Calculation of light delay for coupled microrings by FDTD technique and Padé approximation

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The Padé approximation with Baker’s algorithm is compared with the least-squares Prony method and the generalized pencil-of-functions (GPOF) method for calculating mode frequencies and mode Q factors for coupled optical microdisks by FDTD technique. Comparisons of intensity spectra and the corresponding mode frequencies and Q factors show that the Padé approximation can yield more stable results than the Prony and the GPOF methods, especially the intensity spectrum. The results of the Prony method and the GPOF method are greatly influenced by the selected number of resonant modes, which need to be optimized during the data processing, in addition to the length of the time response signal. Furthermore, the Padé approximation is applied to calculate light delay for embedded microring resonators from complex transmission spectra obtained by the Padé approximation from a FDTD output. The Prony and the GPOF methods cannot be applied to calculate the transmission spectra, because the transmission signal obtained by the FDTD simulation cannot be expressed as a sum of damped complex exponentials. © 2009 Optical Society of America

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1. INTRODUCTION

Recently, slow light has attracted great attention because of its potential applications for optical data buffering in optical signal processing, optical interconnection, optical computing, etc [1–3]. Coupled-microring-resonator waveguides and photonic crystal waveguides were investigated to realize slow light in photonic integrated circuits. The transfer matrix method was applied to analyze the light delay in embedded ring resonators [4], and fast light in an overcoupled ultracompact silicon ring resonator with resonance splitting was demonstrated [5]. The finite-difference time-domain (FDTD) technique is a powerful tool for simulating light transmission in complex microstructures [6]. However, the FDTD simulation yields only the time variation of electromagnetic fields, which must be transformed into the frequency domain to obtain the transmission spectrum. The natural selection is the fast Fourier transform (FFT) method, which has resolution inversely proportional to the total persistence time of the FDTD iteration. So a very large FDTD iteration number, which always means a terrible computation task, is required if a high Q mode exists or several degenerate modes have close frequencies. To save the computing time of the FDTD process, the Prony method [7,8], the matrix-pencil method [9,10], and the FFT/Padé approximation method [11] were used to analyze the FDTD output efficiently. Furthermore, the Padé approximation with Baker’s algorithm [12] was found to be a powerful tool for calculating the field spectrum from the time response signal of the FDTD process [13], which could save more FDTD computing time than the FFT/Padé approximation, especially for a cavity with nearly degenerate modes. The Padé approximation with Baker’s algorithm was extensively applied in the simulation of optical microcavities [14], the propagation loss of photonic crystal waveguides with the highest Q factor of the order of 10\(^{11}\) [15], photonic crystal microcavities [16,17], and the transmission characteristics of optical microring filters [18].

Only the amplitude of the intensity spectrum obtained from the late FDTD output that can be expressed as a sum of damped oscillators is required in calculating mode frequencies and mode Q factors for microcavity structures. However, the calculation of the transmission spectrum needs to transform the whole FDTD output including the initial sequence from time domain to frequency domain. The Prony and the GPOF methods cannot be applied for calculating the transmission spectrum from a FDTD output, because the whole FDTD output is not a sequence signal of damped oscillators. Furthermore, the light delay is related to the derivative of the phase shift spectrum with respect to optical frequency, which requires more exact evaluation of the complex transmission spectrum.

In this paper, we apply the Padé approximation to calculate the light delay in coupled microring structures based on the FDTD simulation. The paper is organized as following. The basic formulas are first presented for the Padé approximation, the least-squares Prony method, and the GPOF method in Sections 2–4, respectively; then the intensity spectra, mode frequencies, and mode Q factors obtained by the three methods are compared for coupled optical microdisks in Section 5; and finally the
light delays for embedded microring resonators obtained by the FDTD technique and the Padé approximation with Baker’s algorithm are presented in Section 6.

2. PADÉ APPROXIMATION WITH BAKER’S ALGORITHM

The Padé approximation with Baker’s algorithm serves to approximate an infinite time sequence with a much shorter time sequence. According to the definition of the Padé approximants [12,13], a given power series \( f(z) \) can be approximately expressed as

\[
f(z) = \sum_{n=0}^{\infty} c_n z^n \tag{1}\]

can be approximately expressed as

\[
\sum_{n=0}^{\infty} c_n z^n - \frac{P(z)}{R(z)} = O(z^{M+N+1}), \tag{2}
\]

where

\[
P(z) = \sum_{n=0}^{M} a_n z^n, \]

\[
R(z) = 1 + \sum_{n=1}^{N} b_n z^n.
\]

\( P(z)/R(z) \) is defined as the Padé approximant \([M,N]_f \) of the given power series \( f(z) \). Assuming \( S(n \Delta t) \) is the time response of one of electromagnetic field components, where \( n=0,1,\ldots,\infty \) is the sampling number and \( \Delta t \) is the sampling time interval in the FDTD simulation, we can obtain field spectrum from the FDTD output by the FFT method:

\[
U(\omega, f) = \sum_{n=0}^{\infty} S(n \Delta t) \exp(-i2\pi f n \Delta t). \tag{3}
\]

Similar to (1), we can define a power series

\[
F(z,f) = \sum_{n=0}^{\infty} C_n z^n, \tag{4}\]

with \( C_n = S(n \Delta t) \exp(-i2\pi f n \Delta t) \) and \( F(1,f) = U(\omega, f) \) to approach the field spectrum (3). In real FDTD simulation, we get only a finite time sequence of the field \( S(n \Delta t) \) with \( n=0, \ldots, N \). Assuming \( N \) is an even number and applying the Padé approximant \([N/2,N/2]_f \) to the given power series (4) at \( z=1 \), we can obtain the approximation of \( U(\omega, f) \). According to Baker’s algorithm, the Padé approximant \([N/2,N/2]_f \) can be calculated by

\[
\eta_{2j+2}(z) = \frac{\bar{\eta}_{2j+1} \eta_{2j}(z) - z \bar{\eta}_j \eta_{2j+1}(z)}{\bar{\eta}_{2j+1} \theta_{2j+1}(z) - z \bar{\eta}_j \theta_{2j+1}(z)}, \tag{5}
\]

\[
\eta_{2j+3}(z) = \frac{\bar{\eta}_{2j+2} \eta_{2j+1}(z) - \bar{\eta}_{2j+1} \eta_{2j+2}(z)}{\bar{\eta}_{2j+2} \theta_{2j+2}(z) - \bar{\eta}_{2j+1} \theta_{2j+2}(z)}, \tag{6}
\]

where \( \bar{\eta}_j \) is the coefficient of the highest power of \( z \) in \( \eta_j(z) \). The initial values of the recursion are

\[
\eta_0 = \sum_{n=0}^{N} C_n z^n, \quad \theta_0 = 1.0, \tag{7}
\]

\[
\eta_1 = \sum_{n=0}^{N-1} C_n z^n, \quad \theta_1 = 1.0. \tag{8}
\]

The field amplitude at a given frequency can be calculated through the recursion relation with the given initial values based on the finite time sequence, and then the intensity spectrum can be calculated by

\[
I(f) = \left| \frac{N}{N/2} \right|_{F(1,f)}^2. \tag{9}
\]

After calculating the intensity spectrum over an interested frequency range, we can obtain the mode frequency \( f_0 \) and the mode quality factor from the frequency of the local maximum and the corresponding 3 dB bandwidth \( \Delta f \) of the intensity spectrum by \( Q=f_0/\Delta f \).

In Baker’s algorithm, the numerator and denominator of the Padé approximation are calculated by the recursion relations at the individual frequency. So the calculation of the algorithm is repeated at each frequency for calculating the whole field spectrum. Because \( 1/\Delta t \) is much larger than the frequency of interest, the FDTD output is usually filtered and decimated to obtain a short sequence to save the computing time of the Padé approximation. In the FDTD/Padé approximation, the coefficients \( a_n \) and \( b_n \) in \( P(f) \) and \( R(f) \) of (2) are first determined by fitting \( P(f)/R(f) \) with the limited resolution FFT output [11], and then a high-resolution field spectrum is directly calculated from \( P(f)/R(f) \) with the fitted coefficients \( a_n \) and \( b_n \). So the FFT/Padé approximation requires less computing time than Baker’s algorithm in transforming the time response signal to the frequency domain. However, the Padé approximation with Baker’s algorithm can save more computing time over the FDTD simulation especially for a cavity with nearly degenerate modes [13].

3. LEAST-SQUARES PRONY METHOD

In the least-squares (LS) Prony method, the time response signal is modeled as a linear combination of damped complex exponentials, and then least-squares linear prediction methods are used for obtaining resonant frequencies and damping factors. The following description of the LS-Prony method is taken from [7]. In this method, the time response signal \( S_n = S(n \Delta t) \) is expressed as a sum of damped complex exponentials:

\[
\eta_{2j+2}(z) = \frac{\bar{\eta}_{2j+1} \eta_{2j}(z) - z \bar{\eta}_j \eta_{2j+1}(z)}{\bar{\eta}_{2j+1} \theta_{2j+1}(z) - z \bar{\eta}_j \theta_{2j+1}(z)}, \tag{7}
\]

\[
\eta_{2j+3}(z) = \frac{\bar{\eta}_{2j+2} \eta_{2j+1}(z) - \bar{\eta}_{2j+1} \eta_{2j+2}(z)}{\bar{\eta}_{2j+2} \theta_{2j+2}(z) - \bar{\eta}_{2j+1} \theta_{2j+2}(z)}, \tag{8}
\]
where \( M \) is the number of resonant modes, \( a_k \) is the complex amplitude, \( z_k \) is the complex pole, and \( f_k \) and \( \alpha_k \) are the resonant frequency and the damping factor of the \( k \)th resonant mode, respectively. According to Prony’s method, the time sequence \( S_n \) satisfies the following difference equations:

\[
S_n = - \sum_{k=1}^{M} b_k S_{n-k}, \quad n = M, M + 1, \ldots, N, \tag{13}
\]

where \( b_k \) (\( k = 1, 2, \ldots, M \)) are the coefficients of the polynomial

\[
P(z) = z^M + b_1 z^{M-1} + \ldots + b_M, \tag{14}
\]

which has complex roots \( z_k \) satisfying (11). By appropriately estimating the number of resonant modes \( M \), the coefficients \( b_k \) can be obtained as the least-squares solution of the linear Eqs. (13). Then the complex poles \( z_k \) are solved by \( P(z) = 0 \) under the determined \( b_k \), and finally the resonant frequency and the damping factor are estimated from \( z_k \) as

\[
f_k = -\frac{|\text{Im} (\ln (z_k))|}{2 \pi \Delta t}, \tag{15}
\]

\[
\alpha_k = -\frac{|\text{Re} (\ln (z_k))|}{\Delta t}, \tag{16}
\]

\[
Q_k = \frac{\pi f_k}{\alpha_k}. \tag{17}
\]

Furthermore, we can substitute \( z_k \) into (11) and solve the resulting least-squares problem to estimate \( \alpha_k \), where \( k = 1, 2, \ldots, M \). After obtaining \( z_k \) and \( \alpha_k \), the intensity spectrum of the signal can be calculated by

\[
I(f) = \left( \sum_{k=1}^{M} \frac{a_k}{\sqrt{2 \pi (f - f_k - \alpha_k)}} \right)^2. \tag{18}
\]

It is a problem to choose an appropriate number of resonant modes \( M \) for the LS-Prony method in the practical calculation.

### 4. GENERALIZED PENCIL-OF-FUNCTION METHOD

For the GPOF method, the time sequence \( S_n \) \((n = 0, 1, \ldots, N)\) are also considered as a linear combination of the damped complex exponentials as in (11) and (12). However, the GPOF method transforms the problem of the poles into the generalized eigenvalues of the matrix \([9, 10]\). The GPOF method for extracting complex poles described in [9] is summarized in the following. In the GPOF method, two matrices \( Y_1 \) and \( Y_2 \) with the dimensions of \((N-L) \times L\) are constructed:

\[
Y_1 = [y_0, y_1, \ldots, y_{L-1}], \tag{19}
\]

\[
Y_2 = [y_1, y_2, \ldots, y_L], \tag{20}
\]

where \( L \) is the pencil parameter and \( y_k \) are the vectors of columns defined as \( y_k = [S_1, S_{1+1}, \ldots, S_{1+N-L-1}]^T \). The two matrices can be written as

\[
Y_1 = Z_1 A Z_2, \tag{21}
\]

\[
Y_2 = Z_1 A Z_2^0 Z_2, \tag{22}
\]

where

\[
Z_1(l, m) = z_m^{l-1} (1 \leq m = M, 1 \leq l \leq N - L), \tag{23}
\]

\[
Z_2(m, l) = z_m^{l-1} (1 \leq l \leq L, 1 \leq m = M), \tag{24}
\]

\[
Z_0 = \text{diag}[z_1, z_2, \ldots, z_M], \tag{25}
\]

\[
A = \text{diag}[a_1, a_2, \ldots, a_M]. \tag{26}
\]

To solve the generalized eigenvalues of the matrix pencil problem, we write

\[
Y_1^* Y_2 Z_1 Z_2^0 = Z_1 Y_2^* p_i = z_i p_i. \tag{28}
\]

To compute the pseudoinverse \( Y_1^* \), we can use the singular value decomposition of \( Y_1 \) as follows:

\[
Y_1 = U D^{-1} V^H, \tag{29}
\]

where \( D = \text{diag}[\sigma_1, \sigma_2, \ldots, \sigma_M, \sigma_{M+1}, \ldots, \sigma_{N-L}] \). The superscript “\( H \)” denotes the conjugate transpose of a matrix. The GPOF method is to seek the resonant frequencies and the damping factors by estimating the \( M \) largest singular values in \( D \). The number of poles \( M \) can be estimated from the singular values, \( \sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_M \geq \ldots \geq \sigma_{\min(N-L)} \), since \( \sigma_{M+1} = \ldots = \sigma_{\min(N-L)} = 0 \) for the noiseless data. For data with noise, we can define a parameter \( R_e \) and choose \( \sigma_M / \sigma_1 \leq R_e \) as the rule for estimating the mode number \( M \). However the parameter \( R_e \) needs to be selected appropriately during the practical computation. If \( D_0 \) is the \( M \) largest singular values and the truncated matrices \( U_0 \) and \( V_0 \) are the front \( M \) columns of \( U \) and \( V \), we can write the truncated pseudoinverse \( Y_1^* \) as

\[
Y_1^* = V_0 D_0^{-1} U_0^H. \tag{30}
\]

Substituting (30) into (28) and then left multiplying by \( V^H \) yields

\[
(Z - z_i I) q_k = 0, \quad k = 1, 2, \ldots, M, \tag{31}
\]

\[
Z = D_0^{-1} U_0^H Y_2 V_0, \tag{32}
\]

where \( q_k = V^H p_k \), \( Z \) is a \( M \times M \) matrix, and \( q_k \) and \( z_k \) are just the eigenvectors and eigenvalues of \( Z \), respectively. By solving the eigenvalues of \( Z \), we can obtain the required complex poles. Based on the poles and the reso-
nant frequencies, the damping factors as well as the quality factors and the intensity spectrum can be calculated from (15)–(18).

5. SIMULATION FOR COUPLED OPTICAL MICRODISKS

Two-dimensional coupled microdisks surrounded by air with a radius \( R = 1 \, \mu m \), gap \( g = 0.05 \, \mu m \), and refractive index 3.2 as shown in Fig. 1 are simulated by the FDTD technique with the square cells 10 nm and the time step \( \Delta t = 2.33 \times 10^{-17} \, s \) satisfying the Courant limit. In the FDTD simulation, Gaussian modulated cosine impulse

\[
P(x,y,t) = \exp[-(t-t_0)^2/(2t_w^2)] \cos(2\pi ft)
\]

with center wavelength \( \lambda = 1.874 \, \mu m \) (\( f = 160 \, \text{THz} \)), pulse center time \( t_0 = 1000\Delta t \), and pulse half-width \( t_w = 400\Delta t \) is added to the z-directional electrical field component \( E_z \) at some points inside the microdisks as exciting sources. The sources are set to be symmetrical about the \( X \) and \( Y \) axes to excite the coupled modes with the same symmetry. A FDTD simulation of 217 steps is performed, and the time to excite the coupled modes with the same symmetry. A FDTD simulation of 217 steps is performed, and the time variation of a selected field component at some points inside the microdisks is recorded as the time response signal, i.e., the FDTD output. The last 214 FDTD outputs are plotted in Fig. 3 at small. The spectra obtained by the GPOF method are sensitive to the parameters of \( \text{deci} = 100, 200, \) and 300. The spectra obtained by the LS-Prony method are sensitive to the parameters of \( \text{deci} = 100, 200, \) and 300. We find that the intensity spectra vary greatly with the value of \( R_p \). In Fig. 4, we plot the intensity spectra obtained by the Padé approximation at \( \text{deci} = 3, 4, \) and 5. The spectra obtained by the Padé approximation with the different decimalized rates match very well when the values of \( \text{deci} \) satisfy the sampling theorem.

Some minor peaks at \( \approx 1999 \) and 1743 nm appear in Fig. 3 at \( R_p = 10^{-2}, 10^{-4}, \) and \( 10^{-6} \). We find that the intensity spectra vary greatly with the value of \( R_p \). In Fig. 4, we plot the intensity spectra obtained by the Padé approximation at \( \text{deci} = 3, 4, \) and 5. The spectra obtained by the Padé approximation with the different decimalized rates match very well when the values of \( \text{deci} \) satisfy the sampling theorem.

The \( Q \) factors obtained by the above three methods are listed in Table 1 for the modes at wavelength \( \lambda = 2.066, \) 1.843, and 1.674 \( \mu m \). Because the spectrum obtained by the LS-Prony method at \( M = 100 \) is very poor, we do not list the \( Q \) factor in this case. The \( Q \) factors obtained by the Padé approximation agree very well with those obtained by the LS-Prony method at \( M = 300 \) and the GPOF method at \( R_p = 10^{-4} \) and \( 10^{-6} \). Although the LS-Prony method and the GPOF method also yield good \( Q \) factors under suitable values of \( M \) and \( R_p \), the Padé approximation has the advantage that it does not depend on any optimized parameter besides the length of the FDTD output.

### Table 1

<table>
<thead>
<tr>
<th>( \text{deci} )</th>
<th>( M = 100 )</th>
<th>( M = 200 )</th>
<th>( M = 300 )</th>
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<td>3</td>
<td>( R_p = 10^{-2} )</td>
<td>( R_p = 10^{-4} )</td>
<td>( R_p = 10^{-6} )</td>
</tr>
<tr>
<td>4</td>
<td>( R_p = 10^{-2} )</td>
<td>( R_p = 10^{-4} )</td>
<td>( R_p = 10^{-6} )</td>
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<td>5</td>
<td>( R_p = 10^{-2} )</td>
<td>( R_p = 10^{-4} )</td>
<td>( R_p = 10^{-6} )</td>
</tr>
</tbody>
</table>

In Fig. 2, we plot the intensity spectra obtained by the LS-Prony method at \( \text{deci} = 3 \) and the resonant number \( M = 100, 200, \) and 300.

Some minor peaks at \( \approx 1999 \) and 1743 nm appear in Fig. 3 at \( R_p = 10^{-2}, 10^{-4}, \) and \( 10^{-6} \). We find that the intensity spectra vary greatly with the value of \( R_p \). In Fig. 4, we plot the intensity spectra obtained by the Padé approximation at \( \text{deci} = 3, 4, \) and 5. The spectra obtained by the Padé approximation with the different decimalized rates match very well when the values of \( \text{deci} \) satisfy the sampling theorem.

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![Fig. 1. Schematic diagram of coupled microdisks of radius \( R \) and gap \( g \) between two disks.](image1.png)

![Fig. 2. (Color online) Spectra obtained by the LS-Prony method from the \( 2^{14} \) FDTD output at \( \text{deci} = 3 \) and resonant number \( M = 100, 200, \) and 300.](image2.png)

![Fig. 3. (Color online) Spectra obtained by the GPOF method from the \( 2^{14} \) FDTD output at \( \text{deci} = 5 \) and the parameter \( R_p = 10^{-2}, 10^{-4}, \) and \( 10^{-6} \).](image3.png)
6. SIMULATION OF LIGHT DELAY FOR EMBEDDED MICRORING RESONATORS

In this section, we simulate the light delay of the TM mode for the embedded ring resonators shown in Fig. 5, which was proposed in [4]. The external radius and the length of the straight waveguide of the racetrack ring are 3.926 μm and 2.04 μm, respectively; the radius of the embedded microring is 2.6 μm; the waveguide width is 0.2 μm with a refractive index of 3.2 surrounded by air, and the air gap between the waveguides is 0.2 μm. The structure parameters are adjusted to have the same resonance wavelength for the racetrack resonator and the inside microring with the mode number difference being an odd number for producing an electromagnetically induced transparency (EIT)-like effect [4]. A uniform mesh with cell size of 10 nm and a 50-cell perfectly matched layer absorbing boundary condition [19] are used in the FDTD simulation, and the time step is chosen to be 2.33 × 10⁻¹⁷ s satisfying the Courant condition. A Gaussian modulated cosine impulse (33) with the pulse half width \( t_w = 29 t_0 \) and the pulse center time \( t_0 = 3 t_w \) are used in the following FDTD simulation.

First, the embedded ring resonators without the input and the output waveguides are simulated with a 2¹⁸ FDTD time step, and the last 2¹⁵ step FDTD output is used to calculate the intensity spectra. The intensity spectra obtained by the Padé approximation, the LS-Prony, and the GPOF methods are plotted in Fig. 6 as the solid, dashed, and the dashed–dotted curves, respectively. The parameters \( M = 1300 \) and \( R_s = 10^{-8} \) are chosen for the LS-Prony and the GPOF methods, respectively, to obtain more coincident intensity spectra by the three methods.

The embedded microring resonators have the mode wavelength of ≈1499 nm as shown in Fig. 6 at mode number 37 for the racetrack resonator and mode number 26 for the inside microring satisfying the condition of the EIT-like effect [4]. A late FDTD output after the exciting source can be expressed as a sum of damped complex exponentials for calculating the mode frequencies and the mode Q factors by the LS-Prony and the GPOF methods. However, the whole FDTD output, which cannot be expressed as a sum of damped complex exponentials, is required for calculating the transmission spectrum. So the LS-Prony and the GPOF methods are not suitable for calculating the transmission spectrum for the embedded microring resonators. In what follows, we calculate the time delay by the FDTD technique and the Padé approximation.

The “bootstrapping” technique is used to set the input pulse in the FDTD simulation [6]. Because the input and the output waveguides have the same width and support only the fundamental mode, we can have the same field distributions in the input and the output waveguides far away from the coupling region. So the ratio of Poynting power flux over the output plane to that of the input plane is equal to the ratio of field intensities at the center points of the output plane to the those of the input plane. We first record the input pulse at the input plane versus time as the input FDTD data for an isolated input waveguide, and then record the output pulse at a through plane versus time as the output FDTD data for the whole embedded microring resonators of Fig. 5 under the same input pulse. The complex input and output field spectra are obtained from the recorded FDTD data with the pulse trans-

<table>
<thead>
<tr>
<th>( \lambda ) (nm)</th>
<th>Padé</th>
<th>M</th>
<th>LS-Prony</th>
<th>R_s</th>
<th>GPOF Method</th>
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<td>1069</td>
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<td>10²</td>
<td>1208</td>
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<td>1018</td>
</tr>
</tbody>
</table>
mit time of 6 ps by the Padé approximation, and then the complex transmission spectra \( t(\omega) = A(\omega) \exp(-i\varphi) \) with the amplitude \( A(\omega) \) and the phase shift \( \varphi \) are calculated as the ratio of the output field spectra to the input field spectra. The light time delay spectrum can be calculated from the phase shift spectrum \( \varphi \) as

\[
\tau = \frac{\varphi}{\omega},
\]

where \( \omega \) is the angular frequency and the time variation factor of the field is \( \exp(i\omega t) \).

The transmission spectra \( A^2(\omega) \) and time delay spectra \( \tau \) of the through port obtained by the FDTD technique and the Padé approximation are plotted in Fig. 7(a) and 7(b) as open circles. The peak transmission coefficient is 0.76 and time delay is 28.5 ps at the peak of 1493.65 nm. It should be noted that the peak position is a little smaller than the mode wavelength in Fig. 6 for the embedded microring resonators without the input and output waveguide. We also calculate the transmission and time delay spectra of the through port by a transfer matrix method [20] based on coupling coefficients calculated by the FDTD simulation. The coupling coefficients between the racetrack resonator and input waveguide of 0.15 and that between the two rings of 0.11 are obtained by FDTD simulation before the pulse transmission over one period inside the microrings [18]. The mode wavelength of 1493.65 nm corresponds to mode numbers of 37 and 26 for the racetrack resonator and the embedded inside mi-
corroding, respectively. Assuming the transmission loss of 3 dB/cm, which is larger than the 2.23 dB/cm used in [4], and that the two coupling rings have the same resonance wavelength with the difference of mode numbers an odd number, we calculate the transmission and time delay spectra for the embedded ring resonators by the transfer matrix method. The transmission and time delay spectra obtained by the transfer matrix method are plotted in Fig. 7(a) and 7(b) as solid curves; they agree well with FDTD results. The numerical results indicate that the Padé approximation is a powerful tool to evaluate the light delay from the FDTD output for saving FDTD computing time in modeling complex optical waveguide structures.

Finally, the propagation of a continuous exciting source at the wavelength 1493.65 nm of the transmission peak is simulated, and the transmission coefficients at the through and the drop ports versus time are plotted in Fig. 8. The initial response up to 7 ps is shown in Fig. 8(a) with the steps corresponding to light transmission one period inside the resonator. Because the coupling efficiency is 0.15 between the input waveguide and the racetrack microring, the transmission coefficient of the through port first increases from zero to about 0.85 as the input continuous wave arrives at a time of ~ 0.14 ps. The corresponding field distribution is similar to that in Fig. 9(a) at 0.1 ps. Then the transmission coefficient of the through port gradually decreases to less than 4 × 10^{-3} from the time of 3 ps to 4.2 ps as the mode field sets up in the external racetrack microring as shown in Fig. 9(b) at 3 ps. With the input continuous wave coupled into the embedded inner microring resonator, the transmission coefficient of the through port gradually increases to 0.59 at the time of 80 ps as shown in Fig. 8(b). The transmission coefficient at the drop port first increases quickly to 0.66 at the time of 4.2 ps and then gradually decreases to 0.049 at 80 ps. The near stable field pattern at the time of 90 ps is plotted in Fig. 9(c) with strong mode field patterns in the inner microring and a half of the external racetrack resonator. However, the transmission coefficient 0.59 of the through port at the time of 90 ps is still smaller than the peak value of 0.76 in Fig. 7(a).

7. CONCLUSIONS

We have compared the intensity spectra obtained by the Padé approximation with Baker's algorithm, the LS-Prony method, and the GPOF method from the FDTD output for coupled optical microdisks. By choosing a suitable mode number, the LS-Prony and the GPOF methods can yield almost the same mode frequencies and Q factors as the Padé approximation, but the mode number and spurious modes greatly affect the obtained intensity spectra. The complex transmission spectra obtained by the Padé approximation with Baker's algorithm are applied to calculate the time delay for embedded microring resonators for reducing the computing time of the FDTD technique. The LS-Prony and the GPOF methods are not suitable for calculating the transmission spectrum from the corresponding FDTD output, because the output cannot be expressed as a sum of damped complex exponentials.

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