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Research Note

Study on simulation of spatial correlative earthquake ground motion field*

XIAO-JUN LI(李小军) FENG-XIN ZHAO (赵凤新) YU-XIAN HU (胡聿贤)
Institute of Geophysics, State Seismological Bureau, Beijing 100081, *China*

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There is a common feature about the life-line system engineering with very high importance that is: a structure extended and distributed in plane on a large scale. The earthquake loadings are different at different parts of the system. The earthquake input for the response analyses of the structure system must be a ground motion field. The observational data show that the earthquake ground motion fields are neither completely correlative nor completely uncorrelative random fields. Great progress has made in studies on the simulation of the spatial correlative earthquake ground motions, based on the empirical relations of the correlation function, correlation coefficient, cross power spectral density function or coherence function researchers proposed themselves random models of earthquake ground motion field (Matsushima, 1975; Feng *et al.*, 1981; Hao *et al.*, 1989; Yamazaki *et al.*, 1992; Abrahamson, 1992). In these models, the distance between points is used as the main parameter describing the spatial correlation of earthquake ground motions, but we do not think that the distance is a quantity directly affecting it. Some reasonable explanations are given about the correlation of earthquake ground motions by statistical analysis through the distance as a controlling parameter, because of that for the given earthquake and specified research site, the distance between points in a specified direction indirectly reflects the effects of the wave traveling path and relative orientation between earthquake source and research site. Therefore, at first, suitable parameters should be selected in order to get reasonable model of spatially correlative earthquake ground motion field. In this paper, the basic ideas about the simulating method proposed by the authors recently are systematically introduced. In our method, the distance between points、the earthquake magnitude、the seismic source distance、the difference of the seismic source distances at the spatial points in research site and the relative orientation between source and research site are reasonably taken into consideration. Here the distance between points is introduced only as a parameter controlling the effects of the uncertainty factors, but the source distance and the difference of the source distances are considered as the main parameters controlling the spatial correlation of earthquake ground motions. And also the methods for the consideration of source dimension、local site condition and uncertainty earthquake occurrence are simply introduced in this paper.

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Contribution No. 96A0075, Institute of Geophysics, SSB, China

1 A model of earthquake ground motion field

The spatial correlation of earthquake ground motions is related to the seismic source characteristic, the relative orientation between seismic source and research site, wave traveling path and medium property. For the local inhomogeneous site, it is still closely related to the local site condition, but the relation is very complicated, so the effects of the local site condition on it should be considered separately.

In our model, the earthquake magnitude (M), the seismic source distances (R) of spatial points, the variation of the seismic source distances (ΔR) and the distances between spatial points (d) are introduced to describe the effects of the seismic source、wave traveling path、relative orientation between source and research site and uncertainty factors. The basic idea of the model is as follows:

1) the ground motion field $u(x, y, z, t)$ is described in a discrete form, that is, the ground motion field $u(x, y, z, t)$ is constituted by the ground motions $u_P(t)$ at the spatially discrete points $P(x_P, y_P, z_P)$;

2) the ground motion $u_P(t)$ at each point P is described in Fourier amplitude spectrum $U_P(f)$ and Fourier phase spectrum $\varphi_P(f)$

$$u_P(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} U_P(f) e^{-i(2\pi f t + \varphi_P(f))} 2\pi df \tag{1}$$

3) to reflect clearly the variation of the earthquake ground motions with the spatial variation in a small spatial region, Fourier amplitude spectrum $U_P(f)$ and Fourier phase spectrum $\varphi_P(f)$ are decomposed into two parts: spatial large scale part and spatial small scale part, they are expressed in following relations

$$U_P(f) = U_l(M, R_P, f) + U_s(M, R_O, d_P, \theta_P, f) + \epsilon_{U_l}(M, R_P, f) + \epsilon_{U_s}(M, R_O, d_P, f) \tag{2a}$$

$$\varphi_P(f) = \varphi_P(f_0) + \int_0^f \Delta\varphi_P(f) df \tag{3}$$

$$\Delta\varphi_P(f) = \Delta\varphi_l(M, R_P, f) + \Delta\varphi_s(M, R_O, d_P, \theta_P, f) + \epsilon_{\Delta\varphi_l}(M, R_P, f) + \epsilon_{\Delta\varphi_s}(M, R_O, d_P, f) \tag{4a}$$

Further, the equations (2a) and (4a) are rewritten as

$$U_P(f) = U_l(M, R_P, f) + U_s(M, R_O, \Delta R_P, f) + \epsilon_{U_l}(M, R_P, f) + \epsilon_{U_s}(M, R_O, d_P, f) \tag{2b}$$

$$\Delta\varphi_P(f) = \Delta\varphi_l(M, R_P, f) + \Delta\varphi_s(M, R_O, \Delta R_P, f) + \epsilon_{\Delta\varphi_l}(M, R_P, f) + \epsilon_{\Delta\varphi_s}(M, R_O, d_P, f) \tag{4b}$$

where

$$\Delta R_P = R_P - R_O = (R_O^2 + d_P^2 - 2R_O d_P \cos\theta_P)^{0.5} - R_0 \tag{5}$$

In above relations, f is the engineering frequency, the subscript "O" indicates the reference point in the site, R_O and R_P are the seismic source distances of point O and an arbitrary point P, ΔR_P is the difference of the seismic source distances at point P and point O, d_P is the distance between point O and point P, θ_P is the angle between line OP and line OE (where E means the source position), U_l and $\Delta\varphi_l$ (where l means large-scale) are the statistical relations of Fourier amplitude spectrum and Fourier phase difference spectrum of ground motion field in the case of large-scale spatial variation, U_s and $\Delta\varphi_s$ (where s means small-scale) are those in the case of small-scale spatial variation, ε_{U_l} and $\varepsilon_{\Delta\varphi_l}$ represent respectively the effects of the uncertainty factors on Fourier amplitude spectum and Fourier phase difference spectrum in the case of large-scale spatial variation, ε_{U_s} and $\varepsilon_{\Delta\varphi_s}$ represent those in the case of small scale spatial variation, $\varphi_P(f_0)$ is the phase value corresponding to the reference frequency f_0 for the ground motion at point P.

Because the variation of Fourier amplitude spectrum is not obvious in a small spatial region and the effect of the small variaton on the correlation of ground motions is very samll, the equation (3) may be simply written as

$$U_P(f) = U_l(M, R_P, f) + \varepsilon_{U_l}(M, R_P, f) \tag{6}$$

In our model, the Fourier phase difference spectrum is decomposed into a spatial large-scale part and a spatial small scale part. In the spatial large scale part, the parameter describing the spatial variaton is the seismic source distance R, which is only required to be accurate to 1 km, but in the spatial small scale part, the parameter describing the spatial variaton is the difference of the source distances ΔR (relative to reference point O), which should be accurate to 1 or 10 m. In fact, the spatial small scale part is a supplementary value to the spatial large scale part to increase the accuracy, because the spatial large scale part is not enough in accuracy to describe the variation of earthquake ground motions in a small spatial region. The spatial small scale part $\Delta\varphi_s$ consists of two parts: certainty part and uncertainty part. The certainty part describes the regular variation value of the smooth Fourier phase difference spectra (by frequency) in a small spatial region, and the uncertainty part includes the statistical uncertainty for the statistical analysis of the regular value from the smooth spectra and the uncertainty of the spatial variation of the difference values between the real Fourier phase difference spectrum and the corresponding smooth spectrum.

The empirical relations included in our model may be obtained by the statistical analysis of the earthquake records from ground motion stations or ground motion arrays. The existing statistical empirical relations of the Fourier amplitude spectrum and phase difference spectrum simulating the earthquake ground motions (Zhao, 1992; Lee, *et al.*, 1993; Zhao, *et al.*, 1995) may be considered as the relations in the spatial large scale part, but the relations in the spatial small scale part is those to be obtained by our researches.

2 Property of the model

In our model, the earthquake ground motion field is simulated by the earthquake ground motions at spatially discrete points, which are directly given based on the empirical relations of Fourier amplitude spectrum and phase spectrum. The quantities describing the correlation of the earthquake ground motions (such as cross spectral density function, coherence function) are not directly involved in our model, so it is necessary to analyze the correlation property of the simulated ground motion field. In the earthquake ground motion field by our model, the cross power spectral density function of the ground motions at point J and point K may be expressed approximately as follows:

$$S_{KJ}(if) = S(f)\rho(f)e^{-i\theta(f)} \tag{7}$$

$$S(f) = U_K(f)U_J(f) \tag{8}$$

$$\rho(f) = \sin\left(\frac{\Delta\varphi_K(f) - \Delta\varphi_J(f)}{2}\right)\Big/\left(\frac{\Delta\varphi_K(f) - \Delta\varphi_J(f)}{2}\right) \tag{9}$$

$$\theta(f) = \varphi_K(f) - \varphi_J(f) - \frac{\Delta\varphi_K(f) - \Delta\varphi_J(f)}{2} \tag{10}$$

where i is a unit complex, the subscript "J" and "K" indicate the point J and point K, $U(f)$ is the Fourier amplitude spectrum, $\varphi(f)$ is the Fourier phase spectrum, $\Delta\varphi(f)$ is the Fourier phase difference spectrum. The equations (7)～(10) show that the spatial correlation of ground motions is indirectly reflected in the empirical relations of the spatial variation of Fourier spectra. In equation 7, $\rho(f)$ means the coherency function, and from equation (9) we know that $\rho(f)$ is determined by the variation of Fourier phase difference spectrum in space. In addition, in a small spatial region $\Delta\varphi(M, R_K, f) - \Delta\varphi(M, R_J, f)$ is approximately zero (if ignoring the random uncertainty) so $\rho(f)$ is mainly determined by variation of Fourier phase difference spectrum in the case of small scale spatial variation.

3 Consideration of the earthquake source dimension

In the model introduced above, earthquake source distance R and the variation of the source distances ΔR are used to reflect the variation of the relative orientation between source and discrete points in the research site. Therefore, it is easy to know that the seismic source is regarded as a point source. The assumption of point source is suitable for ths small earthquake or large far-field earthquake but not suitable for the larger near-field earthquake. Therefore, there exists a problem to be considered, but which can be solved by an empirical Green's function method (Zhao, 1992), that is, the main fault of a large earthquake is divided into several elementary faults, at first, regarding each elementary fault as a point source, the earthquake ground motion field corresponding to the elementary fault can be obtained by the model mentioned above, and then the earthquaek ground motion field corresponding to the large earthquake will be obtained by the superposition method.

4 Consideration of the local site condition

The data of the real earthquake ground motions and theoretical analyses show that the effect of local site condition on the earthquake ground motion is very obvious and there exist better statistical relations for the earthquake ground motion field on a rock site than that on a soil site. Therefore, we suggest an indirect method to obtain the earthquake ground motion field on a soli site, that is:

(1) At first, the earthquake ground motion field on the bedrock below the research soil site is simulated.

(2) The earthquake ground motion field on the bedrock below the research soil site is as the earthquake input, and make response analysis of local site by the numerical method (such as the finite element-finite difference method (Li *et al.*, 1993)) to obtain the earthquake ground motions (time-histories) at discrete points, that is, the spatially discrete values of the earthquake ground motion field on the soil site.

5 Consideration of the uncertainty earthquake

The methods introduced above are only suitable for a given earthquake. However it is impossible to know the position of the source and magnitude in predicting the future earthquake. Therefore, how to consider the uncertainty earthquake is another problem to be solved. The basic ideas of considering the problem in our method is as follows:

(1) Regard each probably occurring earthquake as a given earthquake, and simulate the corresponding earthquake ground motion field in engineering site.

(2) Analyze the earthquake response of the structure (system) separately by taking the earthquake ground motion field for each earthquake as the earthquake input, and calcualte response values of the structure (system).

(3) Calculate the synthetic response values of the structure (system) under all uncertainty earthquakes based on the basic ideas of the synthesis probabilistic method.

6 Conclusion

In this paper, a method of simulating the earthquake ground motion field is introduced. The spatially discrete expression and the direct simulation of the spatially discrete earthquake ground motion field by the Fourier spectra (including the amplitude spectrum and phase spectrum), the effects of source dimension and local site condition, and the uncertainty earthquake problem are studied in the method. In this paper, only the basic idea of our method is introduced, the details of our method and the statistical analysis of the empirical relations by using our method will be presented in other papers.

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